



# RECURSOS DIDÁCTICOS

## CUARTO DE SECUNDARIA

## TRIGONOMETRÍA

### SUMATORIAS DE SENOS Y COSENOS

En el presente capítulo aprenderemos a calcular expresiones tales como por ejemplo:

$$\text{Sen}2^\circ + \text{sen}4^\circ + \text{sen}6^\circ + \dots + \text{sen}44^\circ + \text{sen}46^\circ + \text{sen}48^\circ$$

$$\text{Cos}4^\circ + \text{cos}8^\circ + \text{cos}12^\circ + \dots + \text{cos}44^\circ + \text{cos}48^\circ$$

Para eso debemos tener en cuenta los siguientes conceptos previos

#### ● SUCESIÓN NUMÉRICA

$$2, 4, 6, 8, 10, \dots, 30$$

$$3, 6, 8, 12, 15, \dots, 48$$

observación :

$\Sigma$  → Letra griega sigma denota sumatoria

la expresión :  $\sum_{k=1}^n t_k$

se lee : "Sumatoria de los números de la forma  $t_k$  desde 1 hasta n.

#### ● SERIE NUMÉRICA

$$2 + 4 + 6 + 8 + 10 + \dots + 30 = \sum_{k=1}^{15} 2k$$

$$3 + 6 + 9 + 12 + \dots + 48 = \sum_{k=1}^{18} 3k$$

En nuestro caso :

$$\text{Sen}2^\circ + \text{sen}4^\circ + \text{sen}6^\circ + \dots + \text{sen}46^\circ + \text{sen}48^\circ = \sum_{k=1}^{24} \text{sen}(2k)^\circ$$

$$\text{Cos}4^\circ + \text{cos}8^\circ + \text{cos}12^\circ + \dots + \text{cos}44^\circ + \text{cos}48^\circ = \sum_{k=1}^{12} \text{sen}(4k)^\circ$$

Para poder simplificar sumatorias de ese tipo debemos reconocer lo siguiente:

- P ⇒ Primer ángulo
- U ⇒ último ángulo
- n ⇒ número de términos
- r ⇒ razón de la P.A.

#### ● SUMATORIA DE SENOS

$$\text{sen} \alpha + \text{sen}(\alpha + r) + \text{sen}(\alpha + 2r) + \dots + \text{sen}(\alpha + (n-1)r) =$$

$$= \frac{\text{sen} \frac{nr}{2}}{\text{sen} \frac{r}{2}} \cos\left(\frac{P+U}{2}\right)$$

#### ● SUMATORIA DE COSENOS

$$\text{cos} \alpha + \text{cos}(\alpha + r) + \text{cos}(\alpha + 2r) + \dots + \text{cos}(\alpha + (n-1)r) =$$

$$= \frac{\text{cos} \frac{nr}{2}}{\text{cos} \frac{r}{2}} \cos\left(\frac{P+U}{2}\right)$$

En nuestro ejemplo:

$$1. \text{sen}2^\circ + \text{sen}4^\circ + \text{sen}6^\circ + \text{sen}8^\circ + \dots + \text{sen}46^\circ + \text{sen}48^\circ$$

$$\text{donde : } P = 2 \quad U = 48^\circ \quad n = 24 \quad r = 2^\circ$$

Obs.- n ⇒ número de términos.

$$n = \frac{t_n - t_1}{r} + 1 \Rightarrow n = \frac{48 - 2}{2} + 1 \Rightarrow n = 24$$

Ahora :

$$\text{sen}2^\circ + \text{sen}4^\circ + \text{sen}6^\circ + \dots + \text{sen}48^\circ =$$

$$\frac{\text{sen}\left(\frac{24 \times 2^\circ}{2}\right)}{\text{sen}\left(\frac{2^\circ}{2}\right)} \text{sen}\left(\frac{2^\circ + 48^\circ}{2}\right) = \frac{\text{sen}24^\circ}{\text{sen}1^\circ} \text{sen}25^\circ$$

2.  $\cos 4^\circ + \cos 8^\circ + \cos 12^\circ + \dots + \cos 44^\circ + \cos 48^\circ$

Donde :  $P = 4^\circ$   $U = 48^\circ$   $n = 12$   $r = 4^\circ$

$n = \frac{48-4}{4} + 1 \Rightarrow n = 12$

Ahora :

$\cos 4^\circ + \cos 8^\circ + \cos 12^\circ + \dots + \cos 44^\circ + \cos 48^\circ =$

$$\frac{\operatorname{sen}\left(\frac{12 \cdot 4^\circ}{2}\right)}{\operatorname{sen}\left(\frac{4^\circ}{2}\right)} \cos\left(\frac{4^\circ+48^\circ}{2}\right) = \frac{\operatorname{sen}24^\circ}{\operatorname{sen}2^\circ} \cdot \cos 26^\circ$$

Obs.-

$$\sum_{i=1}^n \sec 3ix = \sec 2x + \sec 6x + \sec 9x + \dots + \sec 3nx$$

$$\sum_{i=1}^n \frac{1}{1 + \operatorname{sen}(ix)} = \frac{1}{1 + \operatorname{sen}x} + \frac{1}{1 + \operatorname{sen}2x} + \frac{1}{1 + \operatorname{sen}3x} + \dots + \frac{1}{1 + \operatorname{sen}nx}$$

**PROPIEDADES**

$$\cos \frac{\pi}{n} + \cos \frac{3\pi}{n} + \cos \frac{5\pi}{n} + \dots + \cos \frac{(n-2)\pi}{n} = \frac{1}{2}$$

$$\cos \frac{2\pi}{n} + \cos \frac{4\pi}{n} + \cos \frac{6\pi}{n} + \dots + \cos \frac{(n-1)\pi}{n} = -\frac{1}{2}$$

$\forall n \in \mathbb{Z}^+ ; \text{ impar } > 1$



¡Ahora hazlo tú!



Ejercicios de Aplicación

1. Reducir :

$E = \operatorname{sen}2^\circ + \operatorname{sen}4^\circ + \operatorname{sen}6^\circ + \dots + \operatorname{sen}178^\circ$

- a)  $\operatorname{tg}1^\circ$                       b)  $\operatorname{tg}2^\circ$                       c)  $\operatorname{ctg}1^\circ$
- d)  $\operatorname{ctg}2^\circ$                       e)  $\operatorname{tg}4^\circ$

2. Simplificar:

$E = \operatorname{sen}10^\circ + \operatorname{sen}20^\circ + \operatorname{sen}30^\circ + \dots + \operatorname{sen}110^\circ$

- a)  $\frac{\sqrt{3}\operatorname{sen}50^\circ}{2\operatorname{sen}5^\circ}$                       d)  $\frac{\operatorname{sen}55^\circ}{2\operatorname{sen}5^\circ}$
- b)  $\frac{\sqrt{3}\operatorname{sen}55^\circ}{2\operatorname{sen}5^\circ}$                       e)  $\frac{\operatorname{sen}50^\circ}{2\operatorname{sen}5^\circ}$
- c)  $\frac{\sqrt{3}\operatorname{sen}55^\circ}{\operatorname{sen}5^\circ}$

3. Reducir :

$E = \cos x + \cos 3x + \cos 5x + \dots + \cos 13x$

- a)  $\frac{\operatorname{sen}7x}{2\operatorname{sen}x}$                       b)  $\frac{\operatorname{sen}14x}{2\operatorname{sen}x}$                       c)  $\frac{\operatorname{sen}16x}{2\operatorname{sen}x}$
- d)  $\frac{\operatorname{sen}7x}{\operatorname{sen}x}$                       e)  $\frac{\operatorname{sen}14x}{\operatorname{sen}x}$

4. Reducir:

$E = \frac{\cos 2x + \cos 4x + \cos 6x + \dots + \cos 18x}{\operatorname{sen}2x + \operatorname{sen}4x + \operatorname{sen}6x + \dots + \operatorname{sen}18x}$

- a)  $\operatorname{tg}8x$                       b)  $\operatorname{ctg}10x$                       c)  $\operatorname{tg}10x$
- d)  $\operatorname{ctg}8x$                       e)  $\operatorname{tg}10x$

5. Calcular :  $E = \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7}$

- a) 1                                      b) -1                                      c) 1/2
- d) -1/2                                      e) 2

6. Simplificar:

$E = \cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11}$

- a) 1                                      b) -1                                      c) 1/2
- d) -1/2                                      e) 1/4

7. Calcular:

$$E = \cos \frac{2\pi}{11} + \cos \frac{4\pi}{11} + \cos \frac{6\pi}{11} + \cos \frac{8\pi}{11} + \cos \frac{10\pi}{11}$$

- a) -1/2                      b) 1/2                      c) -3/2  
d) 3/2                        e) 1

8. Calcular:  $E = \cos^2 \frac{\pi}{7} + \cos^2 \frac{2\pi}{7} + \cos^2 \frac{3\pi}{7}$

- a) 5/2                        b) 5/4                        c) 7/2  
d) 7/4                        e) 7/3

9. Calcular :

$$E = \operatorname{sen}^2 \frac{\pi}{21} + \operatorname{sen}^2 \frac{2\pi}{21} + \operatorname{sen}^2 \frac{3\pi}{21} + \dots + \operatorname{sen}^2 \frac{10\pi}{21}$$

- a) 1                            b) 1/2                        c) 21/2  
d) 21/3                        e) 21/4

10. Reducir:

$$E = \cos 2x + \cos 4x + \cos 6x + \dots + \cos 2nx$$

$$\text{Si : } (n+1)x = \pi$$

- a) 1                            b) -1                        c) -ctgx  
d) -tgx                        e) ctgx

11. Calcular:  $E = \sum_{n=1}^4 \operatorname{sen}^2 \frac{n\pi}{9}$

- a) 9/2                        b) 9/4                        c) 9/8  
d) 7/8                        e) 7/4

12. Reducir:  $E = \sum_{k=1}^n \operatorname{sen} \frac{k\pi}{3}$

- a)  $2 \operatorname{sen} \frac{n\pi}{6} \operatorname{sen}(n+1) \frac{\pi}{6}$   
b)  $2 \operatorname{sen} \frac{n\pi}{6} \cos(n+1) \frac{\pi}{6}$   
c)  $2 \operatorname{sen} \frac{n\pi}{3} \operatorname{sen}(n+1) \frac{\pi}{6}$   
d)  $\operatorname{sen} \frac{n\pi}{2}$   
e)  $\cos \frac{n\pi}{4} \operatorname{sen} \frac{n\pi}{3}$

13. Si :  $\sum_{i=1}^n \left( \frac{1}{1 + \operatorname{tgix}} \right) = a$                        $\sum_{i=1}^n (\operatorname{sec} 2ix) = b$

Hallar :  $E = \sum_{i=1}^n \left( \frac{1}{1 - \operatorname{tgix}} \right)$

- a) n+b+a                      b) n+b-a                      c) n+a-b  
d) n+a                        e) n+b

14. Calcular

$$E = \cos \frac{2\pi}{11} + 2 \cos \frac{4\pi}{11} + 3 \cos \frac{6\pi}{11} + \dots + 10 \cos \frac{20\pi}{11}$$

- a) -9/2                        b) -11/2                        c) 11/2  
d) 9/2                        e) 11

15. Calcular:

$$E = \cos^3 1^\circ + \cos^3 3^\circ + \cos^3 5^\circ + \dots + \cos^3 59^\circ$$

- a)  $\frac{3\sqrt{3}}{4} \operatorname{csc} 1^\circ$                       b)  $\frac{\sqrt{3}}{4} \operatorname{csc} 1^\circ$                       c)  $3\sqrt{3} \operatorname{csc} 1^\circ$   
d)  $4\sqrt{3} \operatorname{sec} 1^\circ$                       e)  $3 \operatorname{sec} 2^\circ$



1. Reducir:

$$E = \operatorname{sen} 4^\circ + \operatorname{sen} 8^\circ + \operatorname{sen} 12^\circ + \dots + \operatorname{sen} 176^\circ$$

- a)  $\operatorname{tg} 2^\circ$                         b)  $\operatorname{sen} 2^\circ$                         c)  $\operatorname{tg} 88^\circ$   
d)  $\operatorname{ctg} 88^\circ$                         e)  $\operatorname{tg} 28^\circ$

2. Reducir:

$$E = \operatorname{sen} x + \operatorname{sen} 3x + \operatorname{sen} 5x + \operatorname{sen} 7x + \operatorname{sen} 9x$$

- a)  $\operatorname{sen}^2 5x \operatorname{sen} x$                       d) 1/2  
b)  $\operatorname{sen}^2 5x \operatorname{csc} x$                       e) 1  
c)  $\operatorname{sen}^2 5x \operatorname{sec} x$

3. Simplificar :

$$E = \operatorname{sen} x + \operatorname{sen} 3x + \operatorname{sen} 5x + \dots + \operatorname{sen} 15x$$

Sabiendo que:  $\operatorname{sen} 2x = \operatorname{sen}^2 8x \cos x$

- a) 2                      b) 1/2                      c) 4  
d) 1/4                      e) 1

4. Calcular:  $E = \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7}$

- a) 1                      b) -1                      c) 1/2  
d) -1/2                      e) -2

5. Calcular :

$$E = \cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11}$$

- a) 1                      b) -1                      c) 1/2  
d) -1/2                      e) -2

6. Reducir:

$$E = \cos \frac{\pi}{9} + \cos \frac{3\pi}{9} + \cos \frac{5\pi}{9} + \cos \frac{7\pi}{9}$$

- a) 1                      b) 2                      c) 1/2  
d) -1/2                      e) 1/4

7. Calcular :  $E = \cos \frac{\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{5\pi}{7}$

- a) 1                      b) -1                      c) 1/2  
d) -1/2                      e) -2

8. Calcular :

$$E = \cos \frac{2\pi}{13} + \cos \frac{4\pi}{13} + \cos \frac{6\pi}{13} + \dots + \cos \frac{12\pi}{13}$$

- a) 1                      b) -1                      c) 1/2  
d) -1/2                      e) -2

9. Calcular :  $E = \sum_{n=1}^8 \cos^2 \frac{n\pi}{17}$

- a) 15                      b) 15/2                      c) 15/4  
d) 17/2                      e) 17/4

10. Calcular:  $E = \operatorname{sen}^2 \frac{\pi}{7} + \operatorname{sen}^2 \frac{2\pi}{7} + \operatorname{sen}^2 \frac{3\pi}{7}$

- a) 7/2                      b) 7/4                      c) 7/8  
d) 7/16                      e) 7/5

11. Calcular :

$$E = \operatorname{sen}^2 x + \operatorname{sen}^2 2x + \operatorname{sen}^2 3x + \dots + \operatorname{sen}^2 mx$$

Si :  $(m + 1)x = \pi$

- a) m/2                      b) m                      c) (m-1)/2  
d) (m+1)/2                      e) (m-2)/2

12. Calcular :

$$E = \cos^2 \frac{\pi}{9} + \cos^2 \frac{2\pi}{9} + \cos^2 \frac{3\pi}{9} + \cos^2 \frac{4\pi}{9}$$

- a) 9/2                      b) 9/4                      c) 7/4  
d) 7/2                      e) 9/8

13. Calcular el valor de:

$$E = \operatorname{sen}^2 \alpha + \operatorname{sen}^2 2\alpha + \operatorname{sen}^2 3\alpha + \dots + \operatorname{sen}^2 (n\alpha)$$

Si :  $\alpha = \frac{2\pi}{n+1}$   $n \in \mathbb{Z}$

- a) n + 1                      b) n + 2                      c) (n+1)/2  
d) (n-2)/3                      e) 1/2

14. Hallar el valor de :

$$E = \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} + \cos \frac{4\pi}{7} \cos \frac{6\pi}{7} + \cos \frac{6\pi}{7} \cos \frac{2\pi}{7}$$

- a) 2                      b) 1                      c) 1/2  
d) -1/2                      e) 1/3

15. Hallar la suma de los "n" primeros términos de:

$$E = \frac{1}{\cos x + \cos 3x} + \frac{1}{\cos x + \cos 5x} + \frac{1}{\cos x + \cos 7x} + \dots$$

- a) 1                      d)  $\frac{\operatorname{sen} x}{\operatorname{sen} 2x \operatorname{sen} (n+1)x}$   
b) 1/2                      e)  $\frac{\operatorname{sen} x}{\cos 2x \cos (n+1)x}$   
c)  $\frac{\operatorname{sen} x}{\operatorname{sen} 2x \cos (n+1)x}$