RECURSOS DIDÁCTICOS

CUARTO DE SECUNDARIA — ÁLGEBRA

FUNCION CUADRATICA Y DESPLAZAMIENTOS

FUNCIÓN CUADRÁTICA

Es una función con dominio en el conjunto de los números reales y cuya regla de correspondencia es:

$$f(x) = ax^2 + bx + c;\ a, b, c \in R;\ a \neq 0$$

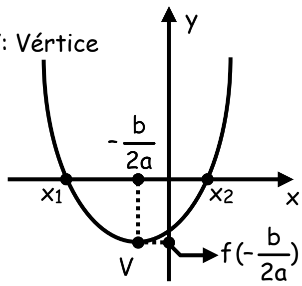
Su gráfica es una parábola simétrica respecto a una recta vertical, llamada eje de simetría, abierta hacia arriba si: $a > 0$ y hacia abajo si: $a < 0$.

Nota gráfica:

Sea la función: $y = ax^2 + bx + c$

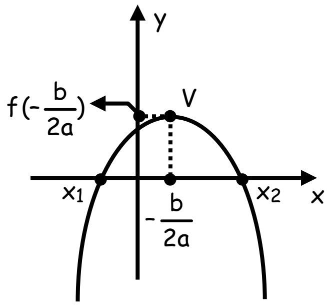
Δ = Discriminante = $b^2 - 4ac$

V: Vértice



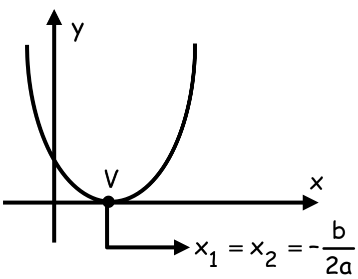
$a > 0 \ \wedge \ \Delta > 0$

$a < 0 \ \wedge \ \Delta > 0$



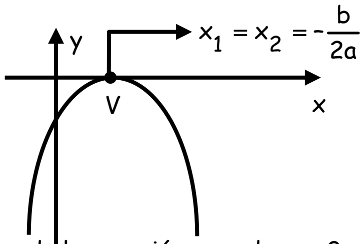
V: Vértice

$\{x_1; x_2\}$ raíces de la ecuación, cuando: $y = 0$

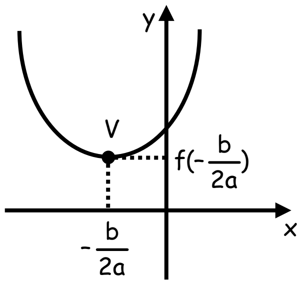


$a > 0 \ \wedge \ \Delta = 0$

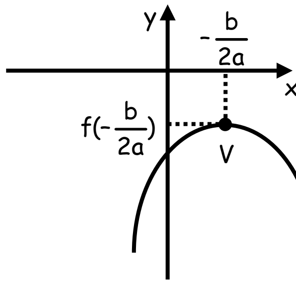
$a < 0 \ \wedge \ \Delta = 0$



$\{x_1; x_2\}$ raíces iguales de la ecuación, cuando: $y = 0$



$a > 0 \ \wedge \ \Delta < 0$



$a > 0 \ \wedge \ \Delta < 0$

Esta función, cuando: $y = 0$, los valores de "x" son números complejos.

OTRAS FUNCIONES

✸ Funciones Pares

Son aquellas funciones que se caracterizan por ser simétricas del eje "y"; y se cumple que:

I. Si: $x \in D_f \rightarrow -x \in D_f$

II. $f(x) = f(-x) \rightarrow \forall\ x \in D_f$

✸ Funciones Impares

Son aquellas funciones que se caracterizan por ser simétricas respecto del origen.

Ejemplo

Indicar que funciones son pares, impares o ni par ni impar:

I. $F_{(x)} = x^4 + 1$

II. $G_{(x)} = x^3$

III. $H_{(x)} = x - |x|$

Solución:

I. $F_{(x)}$ es par, porque:

$F_{(-x)} = (-x)^4 + 1$

$F_{(-x)} = x^4 + 1$

$F_{(-x)} = F_{(x)} \rightarrow \therefore F_{(x)}$ es par

II. $G_{(x)}$ es impar, porque:

$G_{(-x)} = (-x)^3$

$G_{(-x)} = -x^3$

$-G_{(-x)} = x^3$

$-G_{(-x)} = G_{(x)} \rightarrow \therefore G_{(x)}$ es impar

III. $H_{(-x)} = -x - |x|$

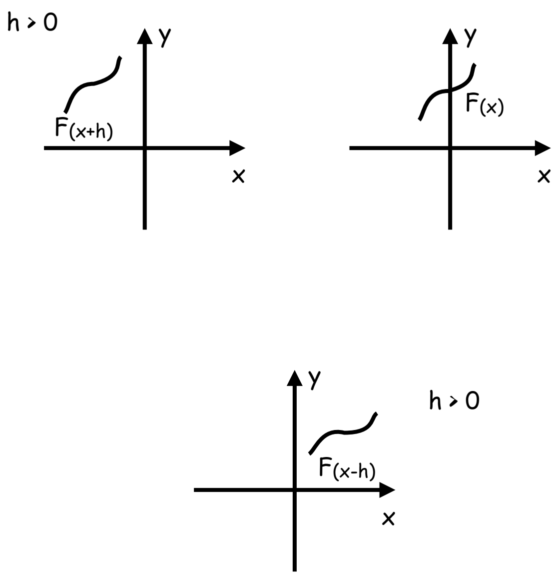
$-H_{(-x)} = x + |x|$

$-H_{(-x)} \neq H_{(x)}$; también: $H_{(-x)} = H_{(x)}$

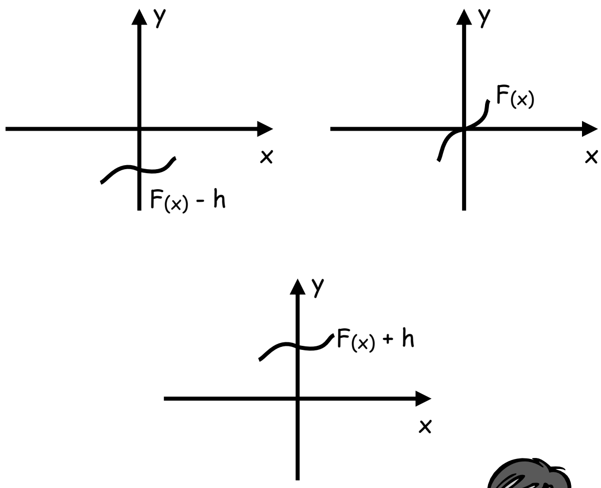
$\therefore H_{(x)}$ no es par ni impar

DESPLAZAMIENTOS

a Desplazamiento Horizontal

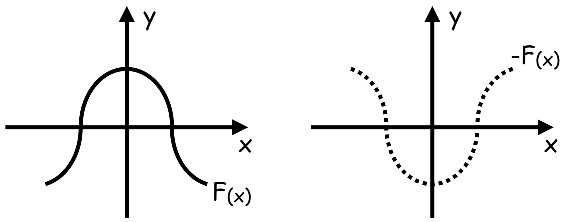


b Desplazamiento Vertical

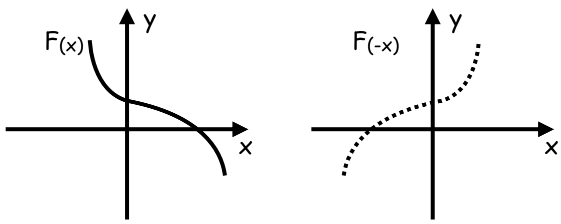


REFLEJOS

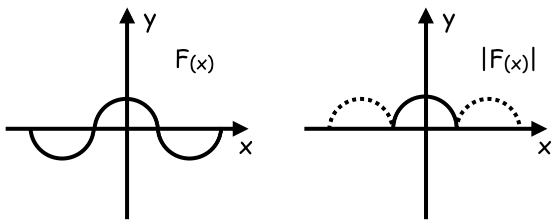
a Reflejo en el Eje x



b Reflejo en el Eje y



c Con valor absoluto



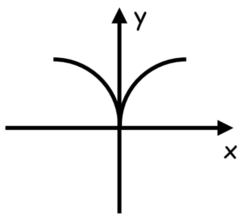
Problemas Resueltos

1. ¿Cuál es la gráfica de: $F_{(x)} = \sqrt{|x|}$?

Solución:

Si: $x \geq 0 \rightarrow \sqrt{|x|} = \sqrt{x} \rightarrow \therefore F_{(x)} = \sqrt{x}$ es la función raíz cuadrada.

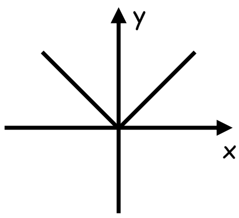
Si: $x < 0 \rightarrow \sqrt{|x|} = \sqrt{-x} \rightarrow \therefore F_{(x)} = \sqrt{-x}$ simétrica a: $\sqrt{x}$ con respecto al eje y de las dos condiciones.



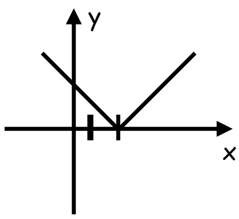
2. Indicar la gráfica de: $F_{(x)} = 7 - |x - 2|$

Solución:

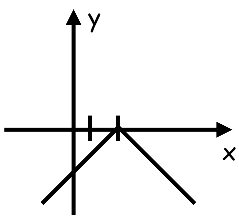
Gráfica 1: $y = |x|$ (función valor absoluto).



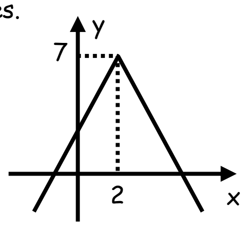
Gráfica 2: $y = |x - 2|$ se desplaza dos unidades a la derecha respecto a $y = |x|$.



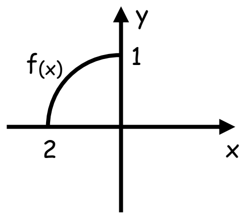
Gráfica 3: $y = -|x - 2|$ es simétrica a: $y = |x - 2|$ con respecto al eje x.



Gráfica 4: $y = 7 - |x - 2|$ se desplaza hacia arriba 7 unidades.



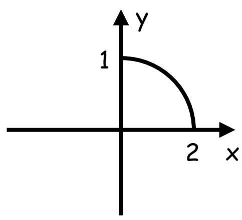
3. Según el gráfico de $f_{(x)}$



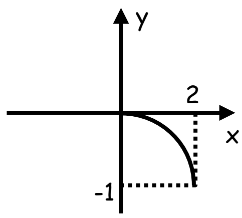
Indicar el gráfico de: $f_{(-x)} - 1$

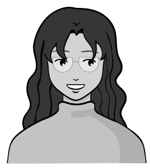
Solución:

$y = f_{(-x)}$ es simétrica a $f_{(x)}$ respecto al eje "y".



$y = f_{(-x)} - 1$ se desplaza una unidad hacia abajo.





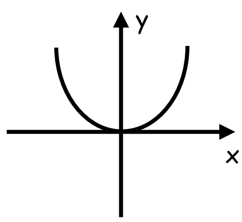
4. Esbozar el gráfico: $F_{(x)} = 4x(x + m) + m^2$ siendo: $m < 0$

Solución:

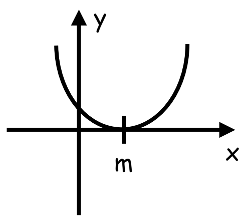
Efectuando:

$$F_{(x)} = \underbrace{4x^2 + 4xm + m^2}_{\text{trinomio cuadrado perfecto}} \rightarrow F_{(x)} = (2x + m)^2$$

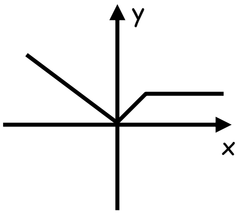
Gráfica 1: $y = (2x)^2 = 4x^2$ (función cuadrática simple)



Gráfica 2: $y = (2x + m)^2$ se desplaza "m" unidades a la derecha, pues: $m < 0$



5. Sea la función $F_{(x)}$ descrita por el gráfico.

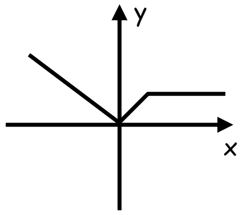


Indicar el gráfico de: $f_{(-x)} - 1$

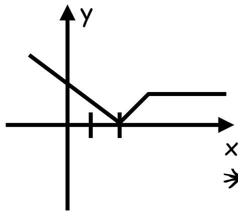
Solución:

Nos piden graficar: $y = F_{(2 - x)} = F_{[-(x - 2)]}$

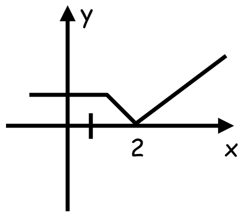
Inicialmente: $y = F_{(x)}$



Gráfica 1: $y = F_{(x - 2)}$. Se desplaza 2 unidades a la derecha.



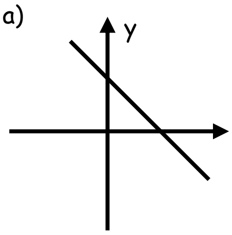
Gráfica 2: $y = F_{[-(x - 2)]}$. Es simétrica en el eje "y" respecto a la función: $y = F_{(x - 2)}$

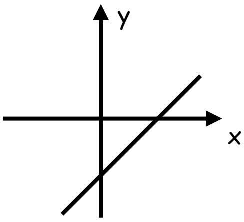


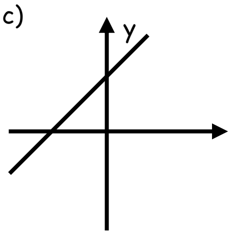
EJERCICIOS DE APLICACIÓN

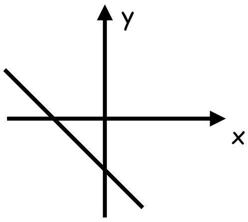
BLOQUE I

1. Graficar: $f_{(x)} = 2x + 3$

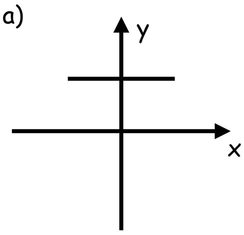
a) 

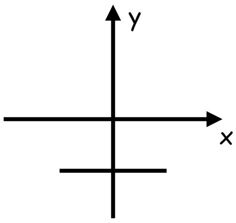
b) 

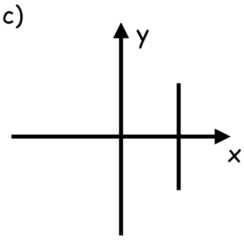
c) 

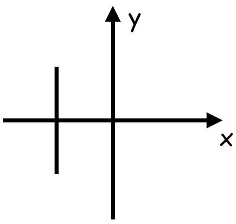
d) 

2. Graficar: $f_{(x)} = -2$

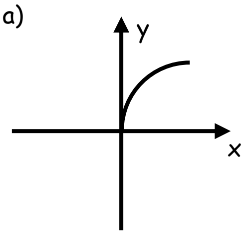
a) 

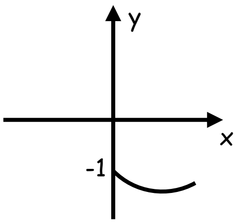
b) 

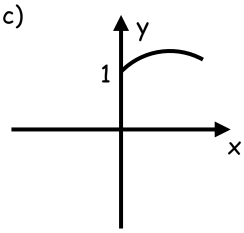
c) 

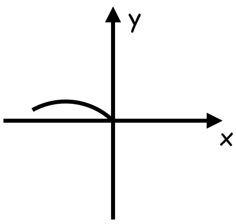
d) 

3. Graficar: $f_{(x)} = \sqrt{x} + 1$

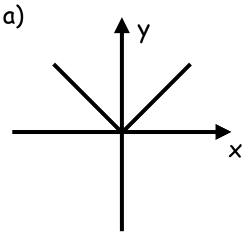
a) 

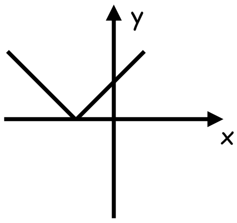
b) 

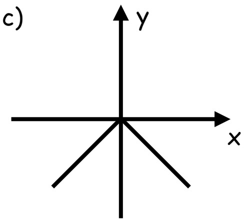
c) 

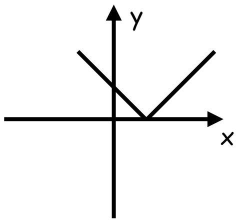
d) 

4. Graficar: $f_{(x)} = |x - 2|$

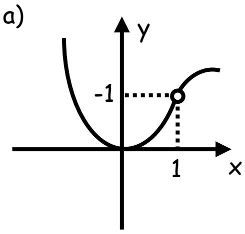
a) 

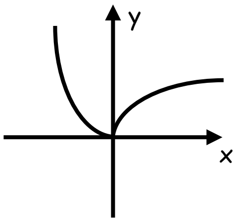
b) 

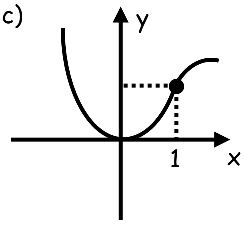
c) 

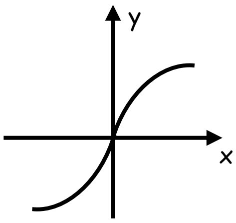
d) 

5. Graficar: $f_{(x)} = \begin{cases} \sqrt{x}; & x \geq 1 \\ x^2; & x < 1 \end{cases}$

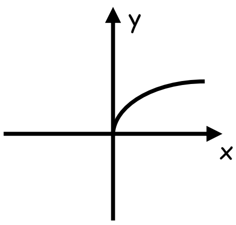
a) 

b) 

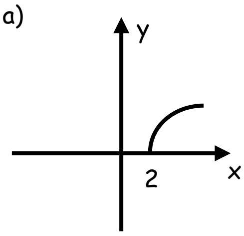
c) 

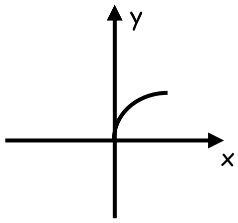
d) 

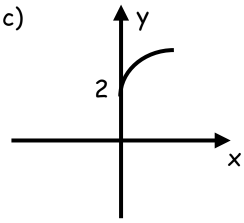
6. Si la gráfica de $f_{(x)} = \sqrt{x}$, es:

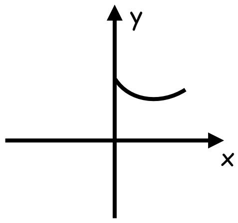


Hallar la gráfica de: $f_{(x)} = \sqrt{x-2}$

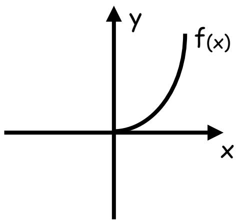
a) 

b) 

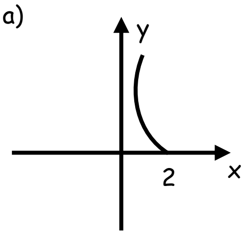
c) 

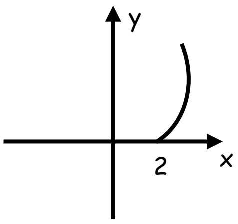
d) 

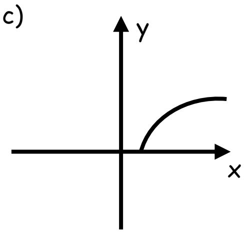
7. Si la función:

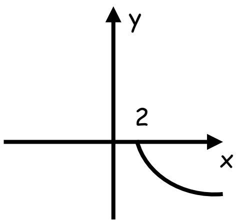


Graficar: $f_{(x-2)}$

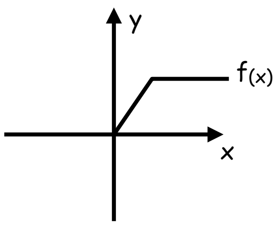
a) 

b) 

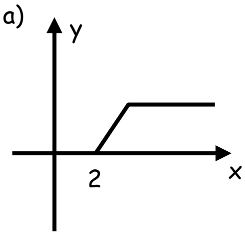
c) 

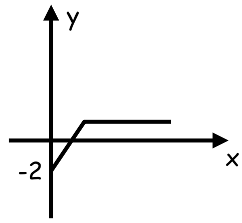
d) 

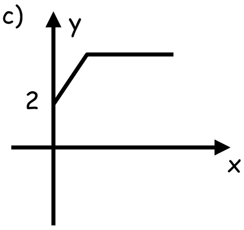
8. Si la función:

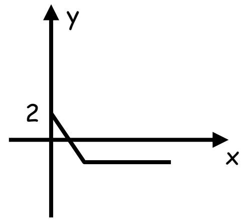


Graficar: $f_{(x)} + 2$

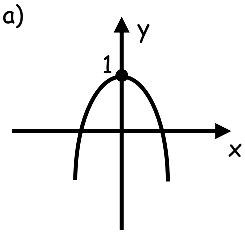
a) 

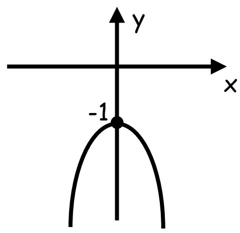
b) 

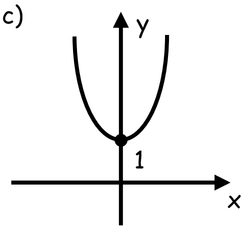
c) 

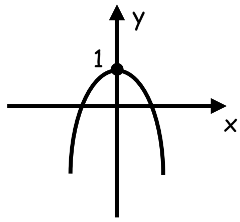
d) 

9. Graficar: $f_{(x)} = x^2 + 1$

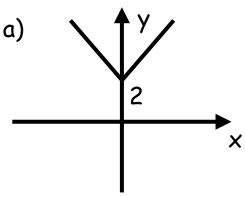
a) 

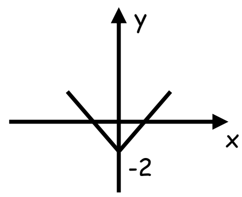
b) 

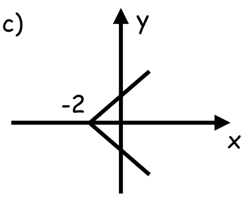
c) 

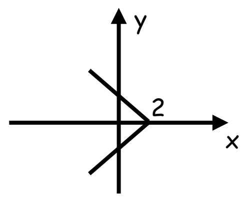
d) 

10. Graficar: $f_{(x)} = |x| + 2$

a) 

b) 

c) 

d) 

11. Sea la función: $F_{(x)} = x^2 + 5x + 1$

Indicar el mínimo valor que toma dicha función.

a) 1 b) 0 c) -1
d) 10 e) 25

12. Para que valor de "x" la función será máxima.

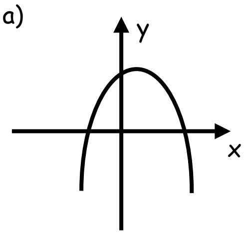
$$f_{(x)} = -x^2 - 25$$

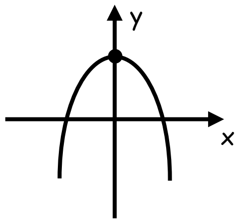
a) 1 b) 25 c) -25
d) 0 e) -1

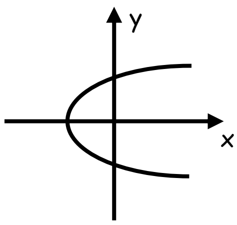
13. Indicar cuál de las siguientes funciones podría ser el gráfico de:

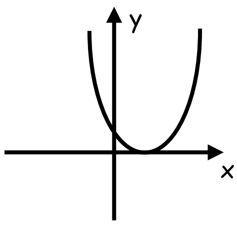
$$f_{(x)} = ax^2 + 3x + 30$$

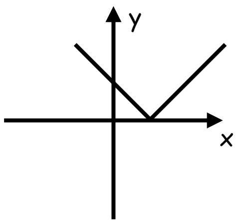
Si: a > 0

a)


b)


c)


d)


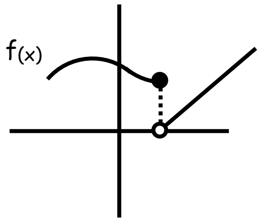
e)


14. Hallar el rango de: $f(x) = 4 - x^2$

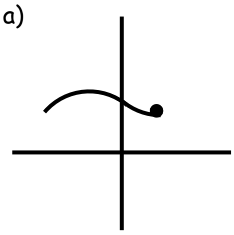
Si: $x \in [-2; 3\rangle$

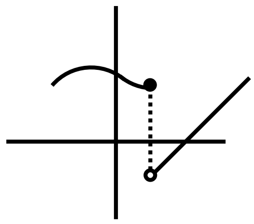
a) $\langle -5; 4\rangle$ b) $\langle -5; 4]$ c) $[-5; 4\rangle$
d) $[-5; 4]$ e) $\langle -\infty; 4\rangle$

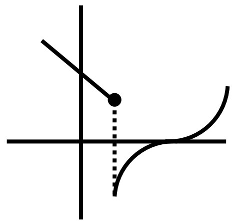
15. Si:

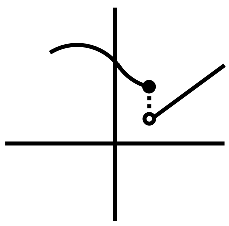


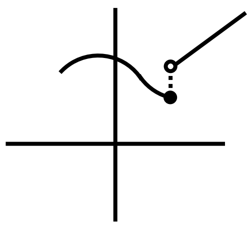
Hallar la gráfica de: $f_{(x)} + 2$

a)


b)


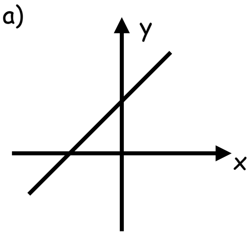
c)


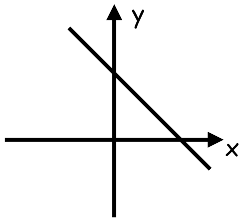
d)


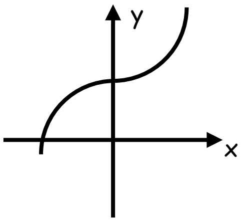
e)


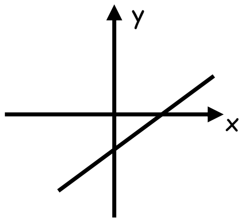
16. Si: $f_{(x)} = x$

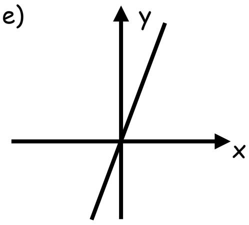
Hallar la gráfica de: $E_{(x)} = f_{(x)} - 3$

a)


b)


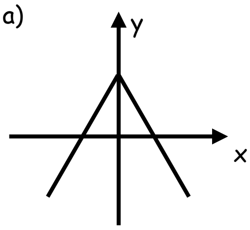
c)


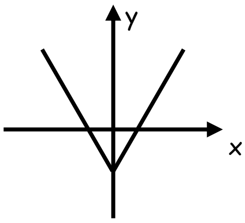
d)


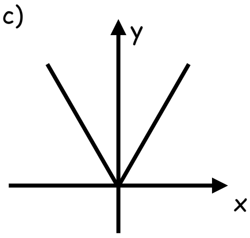
e)


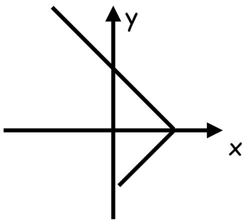
TAREA DOMICILIARIA Nº 8

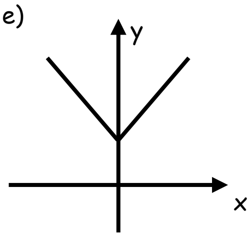
1. Hallar la gráfica de: $f_{(x)} = |x| + 5$

a) 

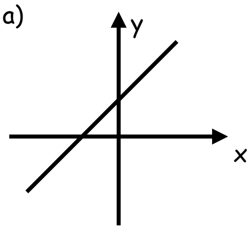
b) 

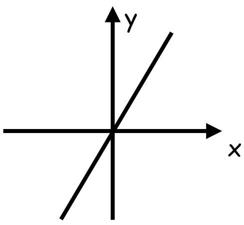
c) 

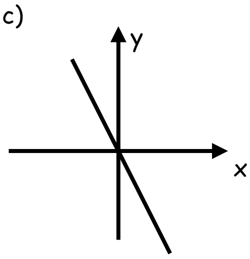
d) 

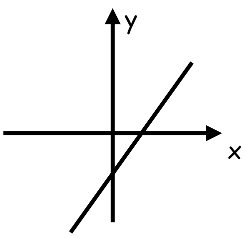
e) 

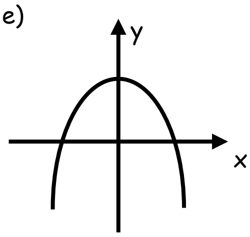
2. Graficar: $f_{(x)} = 3x - 1$

a) 

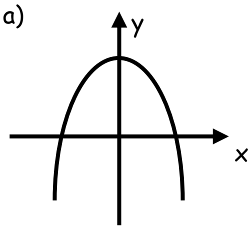
b) 

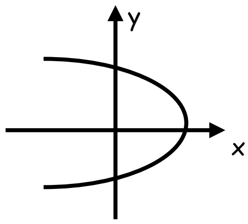
c) 

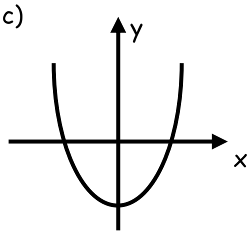
d) 

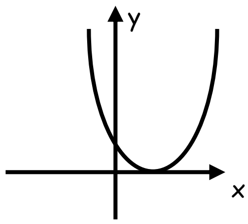
e) 

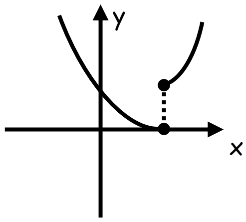
3. Graficar: $f_{(x)} = x^2 - 100$

a) 

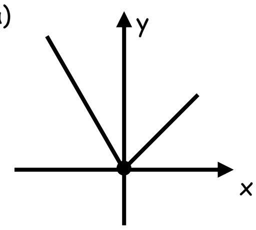
b) 

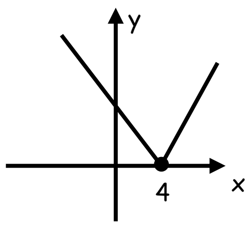
c) 

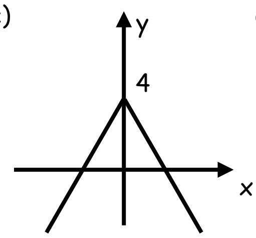
d) 

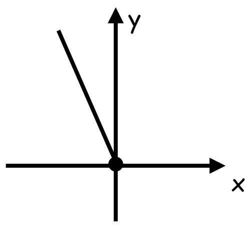
e) 

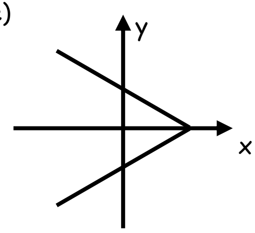
4. Graficar: $f_{(x)} = |x - 4|$

a) 

b) 

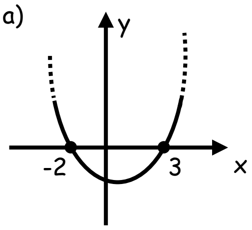
c) 

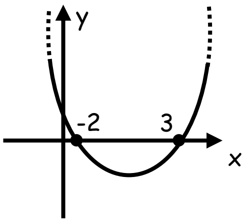
d) 

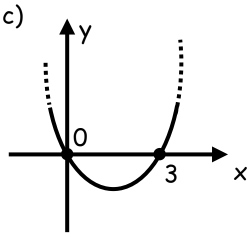
e) 

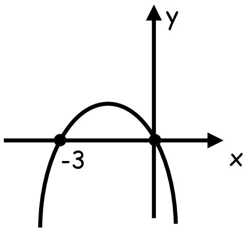
5. Obtener el gráfico de:

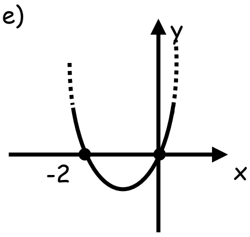
$$y = f_{(x)} = x^2 - 5x + 6$$

a)


b)


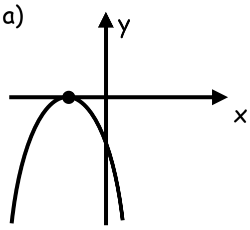
c)


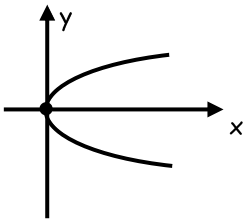
d)


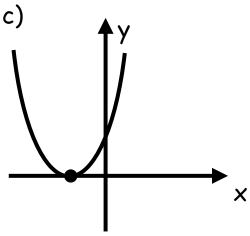
e)


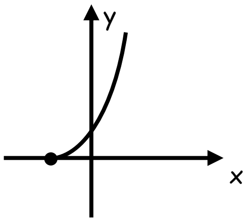
6. Obtener el gráfico de:

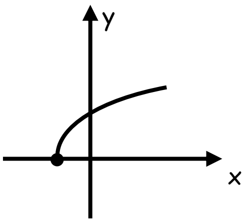
$$y = f_{(x)} = x^2 + 2x + 1$$

a)


b)


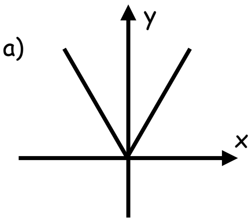
c)


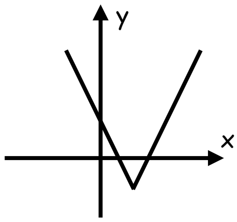
d)


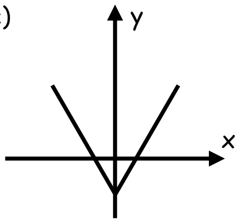
e)


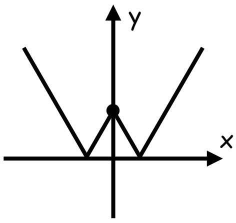
7. Obtener el gráfico de:

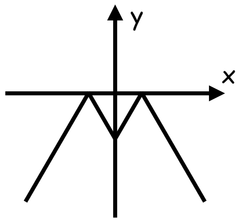
$$F_{(x)} = ||x| - 2|$$

a)


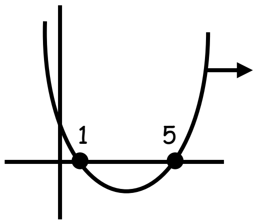
b)


c)


d)


e)


8. Dada la gráfica:



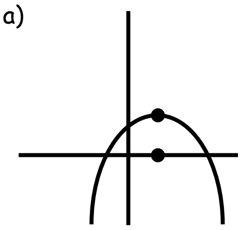
$f_{(x)} = ax^2 + bx + c$

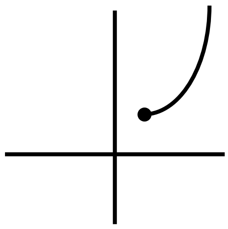
Hallar: a + b + c

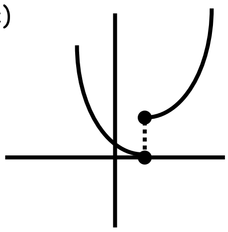
a) 1   b) 2   c) 3
d) 0   e) -1

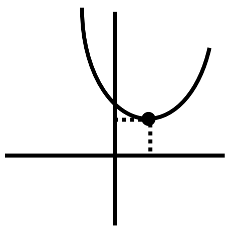
9. Obtener el gráfico de:

$$f(x) = 5(x - 1)^2 + 1$$

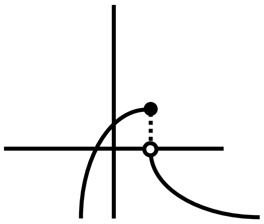
a)


b)


c)


d)


e)



10. Hallar el valor de "x" de manera que la función "f" sea máxima:

$$f(x) = x^2 - 3x + 1$$

a) 3/2 b) -2/3 c) 2/3
d) -3/2 e) 1/3

11. Hallar el valor mínimo que puede tomar la función "f" donde:

$$f(x) = x^2 + 5x + 1$$

a) -21 b) -21/3 c) -21/4
d) 21/4 e) 21/3

12. Hallar el extremo de la función "$f_{(x)}$"
Siendo: $f_{(x)} = -x^2 + 8x + 3$

a) 1 b) 15 c) 16
d) 17 e) 19

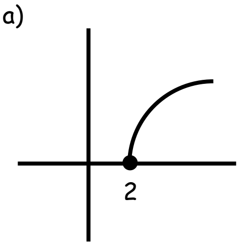
13. Dada la función: $f_{(x)} = 5|x| - 3$
Hallar:

$$E = f(f(-3))$$

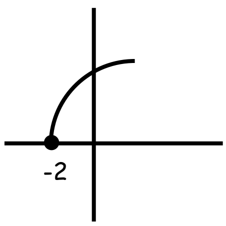
a) 55 b) 56 c) 57
d) 58 e) 59

14. Dada la función: $f_{(x)} = \sqrt{x-2}$
Graficarla.

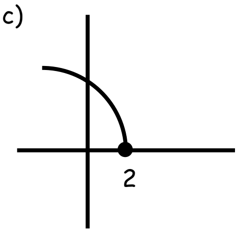
a)



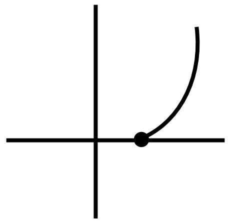
b)



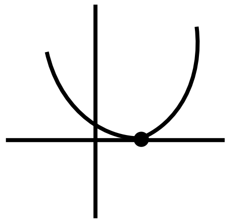
c)



d)



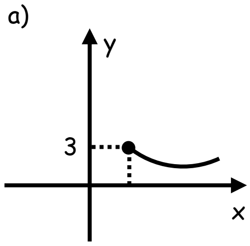
e)



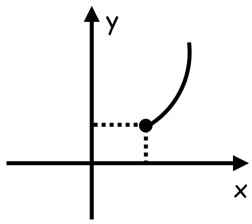
15. Graficar:

$$f_{(x)} = 3\sqrt{x} + 3$$

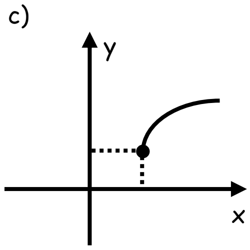
a)



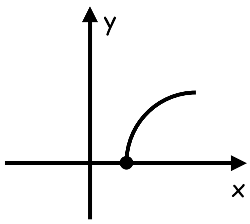
b)



c)



d)



e)